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Signs of Harmonics: Permanent Quad Tuning and Skew Components of Gradient Magnets

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Abstract

The technique for tuning hybrid permanent magnet quadrupoles which is employed for the Recycler Ring Quads[1] employs harmonic measurements of the strength and low order harmonic components. The matrix for tuning was determined empirically. Finding the correct signs is a simple analytic exercise which we will perform as a check on the harmonic measurement results.

1 Introduction

The field $B(r,\theta) = B_y + iB_x$ in a quadrupole magnet is described by

$$B(r,\theta) = B_2 r_0 \sum_{j=1}^{\infty} (b_j + i a_j) \left(\frac{r}{r_0}\right)^{j-1} e^{i((j-1)\theta)} \tag{1}$$

where r and θ are polar coordinates, B_2 is the quadrupole harmonic field, r_0 is the reference radius which we take as 25.4 mm, b_j, a_j are the normal and skew harmonic components with dipole taken as $j = 1$. We report b_j, a_j in 'units' of 1×10^{-4}. Consider $\mathcal{B} = B(r,\theta)e^{i\theta}$

$$\mathcal{B} = B(r,\theta)e^{i\theta} = (B_y + iB_x)(\cos\theta + i\sin\theta) \tag{2}$$

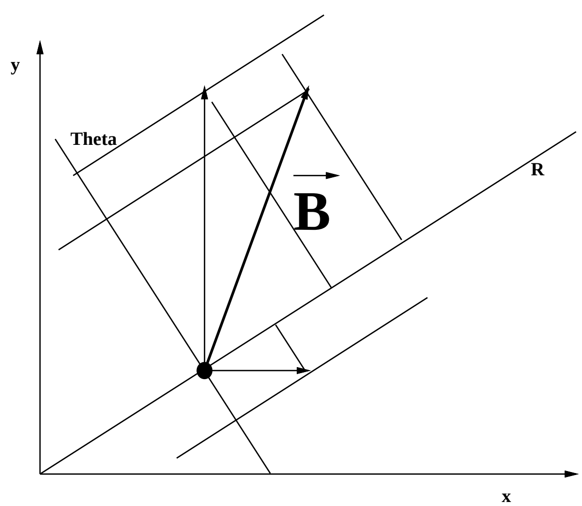

Figure 1: Relations for rectangular and cylindrical coordinates.

$$\mathcal{B} = (B_y \cos\theta - B_x \sin\theta) + i(B_x \cos\theta + B_y \sin\theta) \tag{3}$$

We observe that $\mathcal{B}$ is a complex field related to the cylindrical decomposition for $\vec{B}$ in two dimensions.

$$\mathcal{B} = B_\theta + iB_R \tag{4}$$

where $B_\theta \& B_R$ are the components in a coordinate system at angle θ with respect to the x axis of the usual rectilinear coordinate system. The expansion for $\mathcal{B}$ is therefore

$$\mathcal{B} = B_2 r_0 \sum_{j=1}^{\infty} (b_j + i a_j) \left(\frac{r}{r_0}\right)^{j-1} e^{i(j\theta)}. \tag{5}$$

2 Quadrupole Tuning by Flux Diversion

Reference [1] describes the Recycler Quadrupoles and provides a tuning algorithm. The algorithm has been successfully applied to magnets for production. We propose to check the signs on this tuning algorithm to confirm the signs provided by the existing (and future) harmonics reduction algorithms. The tuning algorithm is described by the equation

$$\begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{pmatrix} = \begin{pmatrix} 0.34 & 1.14 & 1.14 & 13.4 \\ 0.34 & 1.14 & -1.14 & -13.4 \\ 0.34 & -1.14 & -1.14 & 13.4 \\ 0.34 & -1.14 & 1.14 & -13.4 \end{pmatrix} \begin{pmatrix} \delta b_2 \\ b_3 \\ a_3 \\ a_4 \end{pmatrix} \tag{6}$$

where w_1, w_2, w_3, w_4 indicate the changes in the number of washers to be placed behind the pole in quadrant 1-4 respectively. $\delta b_2, b_3, a_3, a_4$ are the measured values of the relative strength error, and the normal and skew sextupole and the skew octupole normalized harmonic components. Two or three adjustment cycles converges to a solution which matches the strength and harmonic requirements. The signs are as reported in Beijing at MT15 and in version of the paper circulated prior to November 20, 1997.

Efforts at MP9 to commission various new measurement hardware has resulted in occasional difficulties with polarities and related issues, and as a result the signs of some of the terms in the array were adjusted, permitting progress in assembly and tuning to continue. Changes in the numbers in the equation are just the result of an independent fit and neither should be assumed to be highly precise. The tuning code as of November 20 can be represented by

$$\begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{pmatrix} = \begin{pmatrix} 0.329 & 1.135 & -1.135 & -13.332 \\ 0.329 & -1.135 & -1.135 & 13.332 \\ 0.329 & -1.135 & 1.135 & -13.332 \\ 0.329 & 1.135 & 1.135 & 13.332 \end{pmatrix} \begin{pmatrix} \delta b_2 \\ b_3 \\ a_3 \\ a_4 \end{pmatrix} \tag{7}$$

where the number of washers to add are calculated so as to make the measured field components become 0 (or at least nearer 0).

3 Flux and Potential Functions from $\mathcal{B}$

Since a convenient complex number representations for magnetic fields and potentials have not been standardized, I will only introduce what I need at this point (remaining consistent with the representations in Reference [2]). We can integrate $\mathcal{B}$ from the origin along a radial line as follows:

$$\int_0^{r_p} \mathcal{B} = \int_0^{r_p} B_\theta + i \int_0^{r_p} B_R = \Phi(r_p, \theta) + iV_m(r_p, \theta) \tag{8}$$

$$\Phi(r_p, \theta) + iV_m(r_p, \theta) = \int_0^{r_p} B_2 r_0 \sum_{j=1}^{\infty} (b_j + ia_j) \left(\frac{r}{r_0}\right)^{j-1} e^{i(j\theta)} \tag{9}$$

$$\Phi(r_p, \theta) + iV_m(r_p, \theta) = B_2 r_0 \sum_{j=1}^{\infty} (b_j + ia_j) e^{i(j\theta)} \int_0^{r_p} \left(\frac{r}{r_0}\right)^{j-1} \tag{10}$$

$$\Phi(r_p, \theta) + iV_m(r_p, \theta) = B_2 r_0^2 \sum_{j=1}^{\infty} (b_j + ia_j) e^{i(j\theta)} \frac{1}{j-1} \left(\frac{r_p}{r_0}\right)^{j} \tag{11}$$

where the integration extends to r_p. We will not be calculating magnitudes so the value taken for r_p will not be important. But its meaning will be obvious in the following.

Two approaches should each give the correct signs for the potential changes required to remove a given field error. First we look at the potential of the pole by examining the potential function at the pole tip (smallest radius). For a normal quadrupole these occur at angles of (45^o, 135^o, 225^o, and 315^o). If we examine the flux function at these angles calling them pole (1, 2, 3, 4) we find

$$V_m(r_p, \theta) = B_2 r_0^2 \sum_{j=1}^{\infty} \frac{1}{j-1} \left(\frac{r_p}{r_0}\right)^{j} (a_j \cos j\theta + b_j \sin j\theta) \tag{12}$$

Considering the 4 poles we can construct the final term in this equation for each as follows (ignoring dipole and $j > 4$): $b_2 + b_3/\sqrt{2} - a_3/\sqrt{2} - a_4, -b_2 + b_3/\sqrt{2} + a_3/\sqrt{2} - a_4, b_2 - b_3/\sqrt{2} + a_3/\sqrt{2} - a_4, -b_2 - b_3/\sqrt{2} - a_3/\sqrt{2} - a_4$ Using this result we can construct the signs for the potential which is to be reduced by increasing the permeance of each pole. We do this by dividing each term by the 1 or -1 which multiplies the b_2 term. The resulting signs, applied to the currently employed matrix, Equation 7, confirms the terms in the present tuning algorithm.

The flux function can also be used to establish the signs of the tuning matrix. Most of the information is available by considering only the flux which crosses the axis at the locations between poles at angles of (360^o, 90^o, 180^o, 270^o). We see that

$$\Phi(r_p, \theta) = B_2 r_0^2 \sum_{j=1}^{\infty} \frac{1}{j-1} \left(\frac{r_p}{r_0}\right)^j (b_j \cos j\theta - a_j \sin j\theta) \tag{13}$$

where we integrate along the axis to the flux return. Again looking only at the values of the final term in the equation we have values on the four axes: $b_2 + b_3 + b_4, -b_2 + a_3 + b_4, b_2 - b_3 + b_4, -b_2 - a_3 + b_4$. We need now consider the net flux into pole 1,2,3, and 4. This is the difference of successive terms on the axes: ($2b_2 + b_3 - a_3, -2b_2 + a_3 + b_3, 2b_2 - b_3 + a_3, -2b_2 - a_3 - b_3$) which again confirms the signs in Equation 7 for the sextupole terms. The normal octapole terms cancel and the skew octapole terms create no flux which crosses the axes.

4 Harmonics in Dipole Symmetry

The field $B(r, \theta) = B_y + iB_x$ in a dipole (or gradient) magnet is described by

$$B(r, \theta) = B_1 \sum_{j=1}^{\infty} (b_j + ia_j) \left(\frac{r}{r_0}\right)^{j-1} e^{i((j-1)\theta)} \tag{14}$$

$$\mathcal{B} = B_1 \sum_{j=1}^{\infty} (b_j + ia_j) \left(\frac{r}{r_0}\right)^{j-1} e^{i(j\theta)} \tag{15}$$

We write the potential function as

$$V_m(r_p, \theta) = B_1 r_0 \sum_{j=1}^{\infty} \frac{1}{j-1} \left(\frac{r_p}{r_0}\right)^j (a_j \cos j\theta + b_j \sin j\theta) \tag{16}$$

Evaluating this at 90^o give for the final term the series $(b_1, -a_2, -b_3, a_4, b_5, -a_6)$ and at 270^o the series is $(-b_1, -a_2, b_3, a_4, -b_5, -a_6)$. We see that the dipole-like (odd) harmonics (j=1,3,5...) have their normal components related to the anti-symmetric (dipole-like) pole potential while the even harmonics (j=2,4,6) have their skew components related to the symmetric (common mode) pole excitation. We note again that a magnet with coils in series has no driving term which is not antisymmetric.

Tuning of the pole potential by moving bricks and compensator will permit one to adjust the skew quadrupole component. This will affect the skew octapole and skew 12-pole also.

References

[1] B. C. Brown, S. M. Pruss, G. W. Foster, H. D. Glass, D. J. Harding, G. P. Jackson, M. P. May, T. H. Nicol, J. F. Ostiguy, P. Schlabach, and J. T. Volk. Hybrid Permanent Magnet Quadrupoles for the Recycler Ring at Fermilab. In *Proceedings of 15th International Conference on Magnet Technology (to be published)*, 1997. Also available as FERMILAB-Conf-97/337.

[2] Henry D. Glass. Measurement of Harmonic Amplitudes and Phases Using Rotating Coils. Technical Report MTF-94-0004 1.1, Fermilab, March 1994.